Problem 2.28

A mass m has speed v_o at the origin and coasts along the x axis in a medium where the drag force is $F(v) = -cv^{3/2}$. Use the "$v\,dv/dx$ rule" (2.86) in Problem 2.12 to write the equation of motion in the separated form $m\,v\,dv/F(v) = dx$, and then integrate both sides to give x in terms of v (or vice versa). Show that it will eventually travel a distance $2m\sqrt{v_o}/c$.

Solution

Apply Newton's second law in the x-direction.

$$\sum F_x = ma_x$$

Let $v_x = v$ to simplify the notation.

$$\begin{aligned} -cv^{3/2} &= m\frac{dv}{dt} \\ &= m\frac{dv}{dx}\frac{dx}{dt} \\ &= m\frac{dv}{dx}v \end{aligned}$$

Solve the differential equation by separating variables.

$$-\frac{c}{m}\,dx = \frac{v}{v^{3/2}}\,dv$$

Integrate both sides definitely, assuming that the velocity of the mass at $x = 0$ is v_o.

$$\int_0^x -\frac{c}{m}\,dx' = \int_{v_o}^v v'^{-1/2}\,dv'$$

$$-\frac{c}{m}x'\bigg|_0^x = 2v'^{1/2}\bigg|_{v_o}^v$$

$$-\frac{c}{m}(x-0) = 2\left(\sqrt{v} - \sqrt{v_o}\right)$$

Therefore,

$$x = \frac{2m}{c}\left(\sqrt{v_o} - \sqrt{v}\right).$$

To find out how far the mass travels before it stops, set $v = 0$.

$$\text{Distance Travelled} = \frac{2m}{c}\left(\sqrt{v_o} - \sqrt{0}\right) = \frac{2m}{c}\sqrt{v_o}$$